Thermocapillary convection in an absorbing–scattering medium subjected to irradiation

Tung T. Lam
The Aerospace Corporation, El Segundo, CA, USA

The criteria for the onset of thermocapillary convection in a horizontal radiating fluid layer heated by an incident thermal radiative energy source are determined. The fluid layer is an absorbing and isotropically scattering medium confined between a free upper surface and an insulated rigid lower surface. Linear analysis is performed on the continuity, momentum, energy, and approximate radiative equations. The resulting disturbance equations are solved using a numerical optimization technique to obtain the eigenvalues governing the onset of convective motion. The influence of thermal radiation on the critical Marangoni number is examined. Attention is drawn to the physical significance of the heat transfer mode, gravitational force, the scattering effect, and the surface radiative properties. The conditions leading to the onset of convection are presented as functions of the optical thickness, scattering albedo, Planck number, surface emissivities, and transmissivities.

Keywords: thermocapillary convection; linear stability analysis; absorbing-scattering medium; optimal control

Introduction

Controlling thermocapillary convection in material processing in space has gained importance as a possible mechanism for producing large crystals of uniform properties and manufacturing materials with unique properties. As the buoyancy force diminishes in the microgravity environment found in many sustained space flights, thermogravitational convection becomes negligible. However, thermocapillary convection generated by variation of surface tension with temperature is a major mechanism that introduces motion in many of these space experiments. Thus, a better understanding of controlled surface-tension-driven convection is important to improve the techniques of material processing in space.

The onset of buoyancy-driven convection in a thermally radiating fluid layer has received considerable attention in the past. For layers with small and large absorption coefficients, Goody (1956) studied the thermal instability of a radiative fluid layer bounded by free surfaces using a variational method. Spiegel (1960) considered the same stability problem for rigid boundaries but neglected the effect of conduction. Murgai and Khosla investigated the combined effects of thermal radiation with rotation (Murgai and Khosla 1962) and magnetic field (Khosla and Murgai 1963) on the Benard problem by following Goody's approach. Christophorides and Davis (1970) extended the work to study the interaction of radiation and conduction. Arpaci and Gozum (1973) studied the thermal instability of fluid layers by using the Eddington approximation for radiation. In a recent investigation, Yang (1990) investigated the interactions of radiating fluids with the surroundings by imposing a radiative heat flux at the free upper surface of the liquid layer. The study was performed on a gray and nonscattering medium.

The studies above are restricted to buoyancy-driven convection, while the effect of surface tension forces in a radiating fluid layer has not been given much attention, although they have a significant impact on many of the space experiments. Bayazitoglu and Lam (1987) studied the thermocapillary convection in a nonscattering medium without external radiation sources and gravitational force. Herein, as an extension of the aforementioned investigation, is a study of the onset of thermocapillary convection in an absorbing–scattering medium subjected to irradiation.

The resulting disturbance equations for the stability problem arise from linear analysis are recast into an optimal control problem. A numerical optimization technique known as the sequential gradient-restoration algorithm is employed to find the eigenvalues governing the onset of convective motion. The critical Marangoni number defining the threshold for the onset of convective instability is determined. Numerical results are presented to illustrate the influences of the incident thermal radiation, scattering albedo, surface radiative properties, and gravitational force.

Outline

A brief description of the physical problem, governing equations, and boundary conditions is first presented. This is followed by the solutions of the base-state temperature and radiative heat-flux profiles. Linear analysis is performed on the governing equations. The resulting disturbance equations are recast into an optimal control problem. The sequential gradient-restoration algorithm is then applied to find the eigenvalues (critical Marangoni and wave numbers) for the convective instability problem. The results are presented for a wide range of thermal and radiative properties, including the Planck number, gravitational force, scattering albedo, and surface properties of the boundaries.

Address reprint requests to Dr. Lam at the Aerospace Corporation, Mail Station M4/916, P.O. Box 92957, Los Angeles, CA 90009-2957, USA.

Received 24 August 1992; accepted 3 April 1993

Mathematical formulation

The physical model considered in this study is depicted in Figure 1. It consists of a radiating incompressible fluid layer of infinite horizontal extent confined between the region $0 \le z \le d$. The origin of the Cartesian coordinate system is affixed on the lower boundary of the fluid layer, and the z-axis is directed vertically upwards. The fluid layer is assumed to be an absorbing and isotropically scattering medium. The lower boundary is a thermally insulated solid surface with hemispherical emissivity and reflectivity of ε_1 and $\rho_1 = 1 - \varepsilon_1$, respectively. The hemispherical reflectivity of the upper free surface is ρ_2, and its transmissivity is τ_{t2}. The upper free surface dissipates heat by convection with a heat transfer coefficient h into an environment at temperature T_∞. Due to an external radiation incidence q'' at the upper free surface, the liquid layer has a nonuniform volumetric energy source.

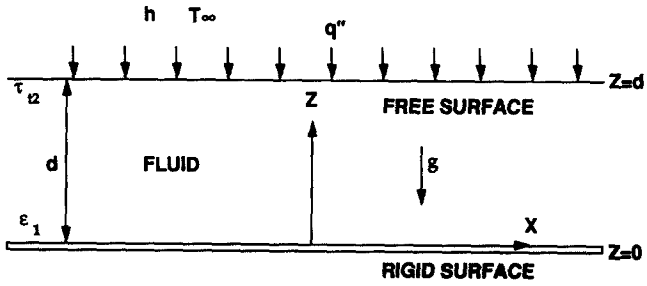

Figure 1 Schematic diagram of the physical system

Governing equations

The physical properties such as viscosity, thermal conductivity, specific heat, and thermal expansion coefficient are independent of temperature, with the exception of the surface tension and the density appearing in the body force (i.e., Boussinesq approximation). The summation convention on repeated indices is used throughout this work, in which the subscripts t, i, and $l(i, l = x, y, z)$ denote partial derivatives with respect to the time and the space coordinates, respectively. By using the Eddington approximation for the equation of radiative transfer, and assuming that the contributions of viscous and radiative stress are negligible, the equations governing the fluid motion are as follows (Arpaci and Gozum 1973; Yang 1990):

Continuity

$$\frac{\partial V_i}{\partial x_i} = 0 \tag{1}$$

Momentum

$$\frac{\partial V_i}{\partial t} + V_l \frac{\partial V_i}{\partial x_l} = \frac{\rho}{\rho_0} g \hat{e}_k - \frac{1}{\rho_0}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 V_i}{\partial x_l^2} \tag{2}$$

Energy

$$\frac{\partial T}{\partial t} + V_l \frac{\partial T}{\partial x_l} = \alpha \frac{\partial^2 T}{\partial x_l^2} - \frac{1}{\rho_0 C_0}\frac{\partial q_l^R}{\partial x_l} \tag{3}$$

Notation

a	Horizontal wave number
Bi	Biot number, hd/k
C_0	Specific heat at constant volume
d	Thickness of the fluid layer
D	Differential operator, d/dZ
$\hat{e}_k$	Unit vector $(0, 0, -1)$
E_b	Blackbody emissive power of the fluid, σT^4
g	Gravitational constant
h	Convective heat transfer coefficient
I	Radiative intensity or functional (Equation 38) to be minimized
j	First moment of radiative intensity, $\int_\Omega I d\,\Omega$
J	Amplitude function of the first moment of radiative intensity
k	Thermal conductivity
Ma	Marangoni number, $[\lvert(\partial\sigma_s/\partial T)_0\rvert d/\alpha\mu_0](q''d/k)$
P	Pressure
Pl	Planck number, $4\sigma T_0^3/(k/d)$
Pr	Prandtl number, ν/α
q''	Incident radiative heat flux
q_l^R	Radiative heat flux
$\bar{Q}_z^R$	Nondimensional base-state radiative heat flux
Ra	Rayleigh number, $(g\beta d^3/\nu\alpha)(q''d/k)$
t	Time
T	Temperature
V_i	Fluid velocity vectors
w	Fluid velocity in the z-direction
W	Amplitude function of the disturbance velocity
x_i, x_l	Spatial coordinates
Z	Nondimensional depth, z/d

Greek symbols

α	Thermal diffusivity, $k/\rho_0 C_0$
α_M	Root mean square absorption coefficient, $\sqrt{(\alpha_P \alpha_R)}$
α_P	Planck mean absorption coefficient
α_R	Rosseland mean absorption coefficient
β	Thermal expansion coefficient
γ_1	$1/[4(1/\varepsilon_1 - 0.5)]$
γ_2	$1/[4(1/\tau_{t2} - 0.5)]$
ε_1	Diffuse emissivity of the lower boundary
η	Degree of nongrayness of fluid, $\sqrt{(\alpha_P/\alpha_R)}$
$\bar{\theta}$	Nondimensional base-state temperature
Θ	Amplitude function of the disturbance temperature
μ	Dynamic viscosity
ν	Kinematic viscosity, μ/ρ
ρ	Density or reflectivity
σ	Stefan–Boltzmann constant
σ_s	Surface tension
τ	Optical thickness, $\alpha_M d$
τ_t	Transmissivity of the upper surface
ω	Scattering albedo
Ω	Solid angle

Superscript

$-$	Base state

Subscripts

0	Reference state
1	Lower boundary
2	Upper boundary
c	Critical
i, l	Spatial index
∞	Environment adjacent to the upper free surface
x, y, z	x, y, and z components

Approximate radiative transfer

$$\frac{\partial^2 j}{\partial x_i^2} - \frac{3\alpha_P\alpha_R}{1-\omega} j = -12\frac{\alpha_P\alpha_R}{1-\omega} E_b \tag{4}$$

The equation of state is defined by assuming a linear density-temperature relation in the form $\rho/\rho_0 = 1 - \beta(T - T_0)$.

Boundary Conditions

The governing equations 1–4 are to be solved subject to the appropriate boundary conditions. The relevant velocity, thermal, and radiative boundary conditions are as follows:

At the lower rigid surface $z = 0$,

$$V_x = V_y = V_z = 0 \tag{5}$$

$$-k\frac{dT}{dz} + q_z^R = 0 \tag{6}$$

$$\gamma_1\frac{\partial q_z^R}{\partial z} - \alpha_p q_z^R = 0 \tag{7}$$

$$\frac{\eta(1-\omega)\alpha_R}{\tau\gamma_1} q_z^R + j = 4E_b \tag{8}$$

At the upper free surface $z = d$,

$$V_z = 0 \tag{9}$$

$$\mu_0\left(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right) = \frac{\partial \sigma_s}{\partial x} \tag{10}$$

$$\mu_0\left(\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y}\right) = \frac{\partial \sigma_s}{\partial y} \tag{11}$$

$$\mu_0\frac{\partial V_z}{\partial z} = -(P - P_0) \tag{12}$$

$$-k\frac{\partial T}{\partial z} = h(T - T_\infty) \tag{13}$$

$$\frac{1}{\alpha_P}\frac{\partial q_z^R}{\partial z} + \frac{1}{\gamma^2} q_z^R = 4[(E_b - E_{b\infty}) - q''] \tag{14}$$

$$\frac{1-\omega}{3\alpha_R\gamma_2}\frac{\partial j}{\partial z} + j = 4(E_{b\infty} + q'') \tag{15}$$

On the upper fluid-free space interface, the differential temperature generates surface tension, and thus induces convection or unstable cellular flows. In the present study, the surface tension σ_S is assumed to vary with the temperature according to $\sigma_s = \sigma_0 + (\partial\sigma_s/\partial T)_0(T - T_0)$.

Base-state analysis

An equilibrium condition exists within the fluid system if the strength of the incident radiative energy source, q'', is sufficiently small. In this study, the aim is to investigate the effect of the incident radiative source q'' on the onset of natural convection in a horizontal liquid layer. Hence, the base-state temperature and the radiative heat-flux profiles within the fluid system are of interest. In the absence of natural convection at the equilibrium state, the equations governing the base-state motions of a liquid layer are (Yang 1990)

$$k\frac{d^2\bar{T}}{dz^2} - \frac{d\bar{q}_z^R}{dz} = 0 \tag{16}$$

$$\frac{d^2\bar{q}_z^R}{dz^2} - \frac{3\alpha_P\alpha_R}{1-\omega}\bar{q}_z^R = 4\alpha_P\frac{dE_b}{dz} \tag{17}$$

subject to the following boundary conditions:

at $z = 0$:

$$-k\frac{d\bar{T}}{dz} + \bar{q}_z^R = 0 \tag{18}$$

$$\gamma_1\frac{d\bar{q}_z^R}{dz} - \alpha_p\bar{q}_z^R = 0 \tag{19}$$

at $z = 1$:

$$-k\frac{\partial\bar{T}}{\partial z} = h(\bar{T} - T_\infty) \tag{20}$$

$$\frac{1}{\alpha_P}\frac{d\bar{q}_z^R}{dz} + \frac{1}{\gamma_2}\bar{q}_z^R = 4[(E_b - E_{b\infty}) - q''] \tag{21}$$

As mentioned previously, the base-state temperature and radiative heat-flux profiles within the fluid layer are required for stability analysis. To solve for the temperature profile, the blackbody-radiation term E_b in the above equations can be linearized, since the high temperature level rather than the large temperature differences are of primary interest.

After scaling the spatial coordinates x_i by d, the time t by d^2/α, the fluid velocity V_i by α/d, the temperature T by $q''d/k$, the pressure P by $\mu\alpha/d^2$, and the radiative heat flux q_i^R, the radiative intensity j, and the blackbody emissive power E_b by q'', the solutions to the nondimensional base-state temperature and radiative heat-flux are

$$\bar{\theta} - \theta_\infty = C_1 + C_2\cosh(mZ) + C_3\sinh(mZ) \tag{22}$$

$$\bar{Q}_z^R = mC_3\cosh(mZ) + mC_2\sinh(mZ) \tag{23}$$

where

$$m = \left(\frac{3\tau^2}{1-\omega} + 4\eta\tau\mathrm{Pl}\right)^{1/2}$$

$$C_1 = \frac{4\left[\left(\frac{m^2}{\eta\tau} + \frac{\mathrm{Bi}}{\gamma_1}\right)\cosh m + m\left(\frac{\mathrm{Bi}}{\eta\tau} + \frac{1}{\gamma_1}\right)\sinh m\right]}{C_4}$$

$$C_2 = -\frac{4\mathrm{Bi}}{C_4\gamma_1}$$

$$C_3 = -\frac{4m\mathrm{Bi}}{C_4\eta\tau}$$

$$C_4 = \frac{m^2}{\eta\tau}\left(4\mathrm{Pl} + \frac{\mathrm{Bi}}{\gamma_1} + \frac{\mathrm{Bi}}{\gamma_2}\right)\cosh m + m\left(\frac{\mathrm{Bi}}{\gamma_1\gamma_2} + \frac{\mathrm{Bi}m^2}{\eta^2\tau^2} + \frac{\mathrm{Pl}}{\gamma_1}\right)\sinh m$$

The stability analysis

The basic governing equations for laminar flow, given by Equations 1–4, are the continuity, momentum, energy, and approximate radiative transfer equations respectively. Based on the conventional linear stability theory, the field variables in these equations are assumed to undergo infinitesimal disturbances. The total instantaneous quantity, which consists of the sum of the steady-state value and a perturbation, can be written as

$$F(X, Y, Z, t) = F_b(Z) + F^*(X, Y, Z, t) \tag{24}$$

where F_b and F^* denote, respectively, the steady-state solution and disturbance quantities for the velocity, temperature, and radiative intensity. The linearized equations governing the initial decay or growth of the disturbances are obtained with the conventional approach (Chandrasekhar 1961) by introducing Equation 24 into the system of Equations 1–4, subtracting the base flow equations, and neglecting all the nonlinear and higher-order terms. To investigate the stability of the fluid system, one can examine the effects of the initial decay or growth of the disturbances. Since there are no lateral boundaries on the system, an infinitesimal two-dimensional (2-D) disturbance can be expressed in the form

$$F^*(X, Y, Z, t) = F(Z) \exp\left[i(a_x X + a_y Y) + \sigma t\right] \tag{25}$$

where $F(Z)$ represents the amplitudes of the radiative intensity, velocity, or temperature; and a_x and a_y are the x- and y-direction wave numbers, which are related to the horizontal wave number by

$$a = (a_x^2 + a_y^2)^{1/2}$$

The complex growth rate σ of the disturbances is given by

$$\sigma = \sigma_r + i\sigma_i$$

The system is at the neutral stable state when $\sigma_r = 0$. For the case $\sigma_r > 0$, the liquid is obviously unstable because the velocity increases exponentially. The base state is said to be stable if σ_r is less than zero.

The principle of exchange of stability has been studied extensively by Davis (1969). In particular, the application of the theorem has been made to Benard convection (Christophorides and Davis 1970; Arpaci and Gozum 1973) with thermal radiation. It has been proven that the principle of exchange of stability holds for convection with radiative transfer for a fluid layer bounded either by an upper rigid or stress-free surface for any optical thickness. On the subject of thermocapillary convection, it has been shown numerically by Vidal and Acrivos (1966) and Takashima (1970) that the principle holds for a fluid layer bounded by an upper undeformable surface. Furthermore, Takashima (1981) has shown that the incipient instability occurs in the stationary mode for surface-tension-driven convection in a horizontal liquid layer with a deformable free surface. This conclusion has also been verified by Castillo and Velarde (1982), who performed a first-order analysis based on Galerkin's method to study the buoyancy-thermocapillary and interfacial deformation effects in one- and two-component fluid layers.

In this general context, the principle of exchange of stability is valid for the present thermocapillary convection problem. The perturbation equations governing the Z-component of the velocity, the temperature, and the radiative intensity perturbations can be formulated by introducing Equation 25 into the resulting linearized equations. Assuming that the validity of the exchange of stabilities holds, the real and imaginary parts of the most unstable eigenvalue vanish at the marginal state that corresponds to the stationary instability. As a result, the perturbation equations governing the marginal state for the Z-component of the velocity, the temperature, and the radiative intensity can be written as (Bayazitoglu and Lam 1987; Yang 1990)

$$(D^2 - a^2)^2 W = a^2 \mathrm{Ra}\Theta \tag{26}$$

$$(D^2 - a^2 - 4\eta\tau \mathrm{Pl})\Theta + \eta\tau J = \frac{d\bar{\theta}}{dZ} W \tag{27}$$

$$\frac{12\tau^2}{1-\omega} \mathrm{Pl}\Theta + \left(D^2 - a^2 - \frac{3\tau^2}{1-\omega}\right) J = 0 \tag{28}$$

with the boundary conditions as follows:

at $Z = 0$:

$$W = 0 \tag{29}$$

$$DW = 0 \tag{30}$$

$$D\Theta + \frac{\eta(1-\omega)}{3\tau} DJ = 0 \tag{31}$$

$$4\mathrm{Pl}\Theta + \frac{\eta(1-\omega)}{3\tau\gamma_1} DJ - J = 0 \tag{32}$$

at $Z = 1$:

$$W = 0 \tag{33}$$

$$(D^2 + a^2)W + \mathrm{Ma}\, a^2 \Theta = 0 \tag{34}$$

$$D\Theta + \mathrm{Bi}\Theta = 0 \tag{35}$$

$$\frac{\eta(1-\omega)}{3\tau\gamma_2} DJ + J = 0 \tag{36}$$

The boundary conditions (Equations 29, 30, and 33) are the familiar boundary conditions of the classical Benard problem. Equations 29 and 33 imply that the normal components of the velocities vanish at both boundaries, $Z = 0$ and $Z = 1$. Equation 30 represents the no-slip, impermeability conditions at the bounding lower rigid surface. Boundary conditions 31, 32, and 36 account for the effects of radiation. The surface tension effect at the free surface, represented by Equation 34, designate the continuity of the tangential stress at the interface. Equation 35 expresses the interfacial thermal condition, and represents the fluid medium that exchanges heat with the environment. The parameters that control the stability include the wave number (a), Biot number (Bi), Marangoni number (Ma), Rayleigh number (Ra), optical thickness (τ), transmissivity of the upper surface (τ_{12}), emissivity of the lower surface (ε_1), scattering albedo (ω), Planck number (Pl), nongrayness parameter (η), and the dimensionless base-state temperature profile ($d\bar{\theta}/dZ$).

The Biot number is the ratio of the convective heat transfer to the internal thermal conduction across the fluid layer. It represents the heat transfer condition at the free surface. The Marangoni number, defined as the ratio of the surface tension to heat diffusion and the viscous force, is a measure of both the surface tension gradient along the free surface and the temperature difference across the layer. The critical Marangoni number, $\mathrm{Ma_C}$, denotes the minimum temperature difference at which instability will occur. The Rayleigh number is a measure of the buoyancy force and the viscous force. The Planck number is the ratio of radiation to conduction. The nongrayness η is defined as the ratio of the optical depths based on the Rosseland mean and the Planck mean absorption coefficients.

The disturbance equations 26–28, together with the homogeneous boundary conditions 29–36 constitute an eigenvalue problem. The nontrivial functions W, Θ, and J satisfy all of these conditions only if there exists a functional relationship such that

$$f(a, \mathrm{Bi}, \mathrm{Ma}, \mathrm{Ra}, \tau, \tau_{12}, \varepsilon_1, \omega, \eta, \mathrm{Pl}, d\bar{\theta}/dZ) = 0 \tag{37}$$

The primary objective of this investigation is to determine the critical value of the Marangoni number and the corresponding wave number on the locus of states that are neutrally stable. The states are represented by the parametric space given by Equation 37. Also, the dimensionless base-state temperature profile $d\bar{\theta}/dZ$ across the liquid layer is examined, and its radiation effect on the thermal convective motion is determined.

Method of solution

Equations 26–36 are the required perturbation equations governing the stability problem under consideration. The eigenvalue problem is recast as an optimal control problem and solved with a numerical optimization technique. The following demonstrates how the resulting perturbation equation for the stability problem can be recast into an optimal control problem and provides a brief discussion of the solution method.

Optimal control problem formulation

The higher-order differential equations (Equations 26–36) governing the stability problem can be rewritten as a system of simultaneous first-order equations. The resulting mathematical problem will then be presented as an optimal control problem.

In the following development, the symbol Z denotes the nondimensional spatial variable; the symbols $x_i(Z), i = 1, \ldots, m$, denote the components of the state (W, J, Θ and their derivatives).

$$\frac{dx_m}{dZ} = \frac{d^m W}{dZ} \qquad m = 1, 2, 3, 4$$

$$\frac{dx_m}{dZ} = \frac{d^{m-4}\Theta}{dZ} \qquad m = 5, 6$$

$$\frac{dx_m}{dZ} = \frac{d^{m-6}J}{dZ} \qquad m = 7, 8$$

With these notations, the differential equations 26–28 can be written as a system of first-order differential equations. Thus, one can recast the convective instability problem presented previously into an optimal control problem in the following form:

Minimize the functional

$$I = \mathrm{Ma} \tag{38}$$

subject to the differential constraints

$$\frac{dx_1}{dZ} = x_2 \tag{39}$$

$$\frac{dx_2}{dZ} = x_3 \tag{40}$$

$$\frac{dx_3}{dZ} = x_4 \tag{41}$$

$$\frac{dx_4}{dZ} = 2a^2x_3 - a^4x_1 + a^2\mathrm{Ra}x_5 \tag{42}$$

$$\frac{dx_5}{dZ} = x_6 \tag{43}$$

$$\frac{dx_6}{dZ} = (a^2 + 4\eta\tau\mathrm{Pl})x_5 - \eta\tau x_7 + \frac{d\bar{\theta}}{dZ}x_1 \tag{44}$$

$$\frac{dx_7}{dZ} = x_8 \tag{45}$$

$$\frac{dx_8}{dZ} = \left(a^2 + \frac{3\tau^2}{1-\omega}\right)x_7 - \frac{12\tau^2\mathrm{Pl}}{1-\omega}x_5 \tag{46}$$

and the boundary conditions

at $Z = 0$:

$$x_1 = 0 \tag{47}$$

$$x_2 = 0 \tag{48}$$

$$x_6 + \frac{\eta(1-\omega)}{3\tau}x_8 = 0 \tag{49}$$

$$4\mathrm{Pl}x_5 + \frac{\eta(1-\omega)}{3\tau\gamma_1}x_8 - x_7 = 0 \tag{50}$$

at $Z = 1$:

$$x_1 = 0 \tag{51}$$

$$x_3 + a^2x_1 + \mathrm{Ma}a^2x_5 = 0 \tag{52}$$

$$x_6 + \mathrm{Bi}x_5 = 0 \tag{53}$$

$$\frac{\eta(1-\omega)}{3\gamma_2}x_8 + x_7 = 0 \tag{54}$$

The eigenvalue problem defined by Equations 38–54 constitutes an optimal control problem. The problem is solved for a given base-state conduction temperature gradient ($d\bar{\theta}/dZ$) across the liquid layer by specifying the values of Bi, a, Ma, Ra, τ_{t2}, ε_1, ω, η, and Pl for various values of the optical thickness τ. The aim of the problem is to minimize the Marangoni number and the corresponding wave number for these particular physical parameters. The sequential gradient-restoration algorithm (SGRA) has been successfully applied to the study of fluid flow (Lam and Bayazitoglu 1986a) and heat transfer (Lam and Bayazitoglu 1986b, 1987, 1988), and is selected as the solution technique for this problem.

Sequential gradient-restoration algorithm

SGRA is a first-order algorithm developed by Gonzalez and Miele (1978) for the optimal solution of mathematical problems involving both inequality and equality constraints. The sequential gradient-restoration algorithm involves a sequence of two-phase cycles, each cycle including the gradient phase and the restoration phase. In the gradient phase, the value of the augmented functional is decreased while avoiding excessive constraint violation. In the restoration phase, the constraint error is decreased while avoiding excessive change in the value of the functional. In a complete gradient-restoration cycle, the value of the functional is decreased while the constraints are satisfied to a predetermined accuracy. Hence, a succession of suboptimal solutions is obtained. The iterative procedure is terminated when both the functional and constraints meet their convergence criteria. The reader is referred to Gonzalez and Miele (1978) for details regarding SGRA. The convergence history of SGRA is illustrated in Figure 2 for a single variable function.

Results and discussion

The conditions leading to the onset of thermocapillary convection in a horizontal participating medium subjected to external radiative incidence have been obtained by using SGRA. The present study focuses only on the marginal stability mode. The critical Marangoni number that defines the threshold for the onset of thermal convective instability has been determined as a function of the Planck number, the optical thickness, the Rayleigh number, the scattering albedo, and the emissivity and transmissivity of the bounding surfaces. The

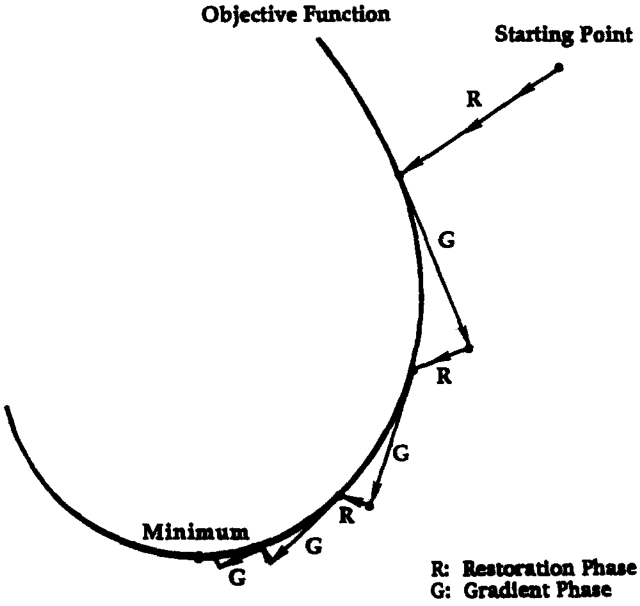

Figure 2 Convergence history of the sequential gradient-restoration algorithm

effects of the upper thermal boundary condition (in terms of Bi) and nongrayness (η) have been previously investigated for a similar system, and therefore their effects will not be re-examined in the present study. Interested readers should refer to an accompanying paper given by Bayazitoglu and Lam (1987) for details.

As a result of the absorbed and scattered thermal radiative energy due to the incident radiative heat flux, the base-state temperature gradient in the conduction regime plays an important role on the conditions leading to the onset of thermal convective motions. The influence of the optical thickness on the base-state temperature profile is shown in Figure 3a. For small optical thickness ($\tau = 0.0001$), this corresponds to a nonparticipating medium; in such a case the incoming energy is absorbed throughout the fluid layer. Therefore, the temperature gradient corresponds to the case of uniform heat generation and deviates from unity with a smooth slope. The same trend continues to hold for moderate small values of τ. However, as τ is increased further, the influence of the optical thickness on the temperature gradients has more profound effects. With increasing optical thickness, the radiation is absorbed mainly in the top strata. As a result, the temperature gradient is steep near the upper boundary and flat near the bottom of the liquid layer. The upper strata are more unstable than the lower region, which has a more uniform temperature.

Figure 3b shows the effect of the scattering albedo on the base-state temperature distribution. The scattering albedo ω is defined as the ratio of the scattering coefficient to the extinction coefficient (sum of the scattering and absorption coefficients) of the medium. For moderate scattering albedo, the incident energy is absorbed uniformly throughout the medium; hence, the temperature gradient changes gradually across the fluid layer. As ω is increased further, the attenuation of the incoming energy process becomes more profound, which decreases the radiation energy reaching the lower boundary. Physically, this corresponds to the situation in which the penetrating radiation energy is absorbed mainly in the top stratum. The change in fluid temperature occurs mainly near the upper boundary and is confined within a small region. Therefore the degree of stability of the system is increased for large ω.

The effects of the lower solid boundary emissivity (ε_1) on the critical Marangoni number are shown in Figure 4. The most unstable situation corresponds to a black surface for $\varepsilon_1 = 1.0$. In this case, all the incoming radiation reaching the lower surface is absorbed and none is reflected. Boundaries of unity emissivity yield the lowest critical values of the Marangoni number. The case of mirror boundary ($\varepsilon_1 = 0$) corresponds to the most stable situation. The radiation is totally reflected by the surface and absorbed wholly within the fluid. As $\tau \gg 1$, most of the penetrating radiation energy is absorbed in the upper stratum and a very small fraction of it reaches the lower surface, and the emissivities of the boundaries do not play a significant role in the stability of optically thick medium. The emissivity of the bounding surfaces are not of major importance because of this substantial attenuation of the incident radiation in the top strata.

In Figure 5, the critical Marangoni number is presented as function of the optical thickness τ for various transmissivities

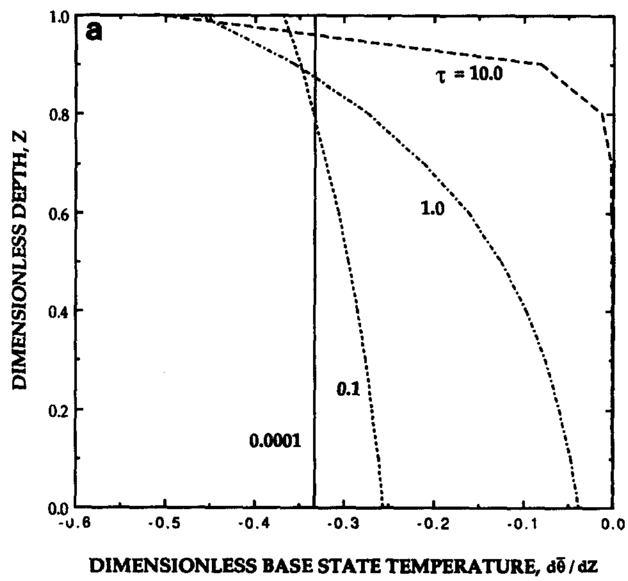

Figure 3a Effect of the optical thickness on the base-state temperature distribution for $\omega = 0$, Bi = 1, Pl = 1, $\eta = 1$, $\varepsilon_1 = 1$, and $\tau_{t2} = 1$

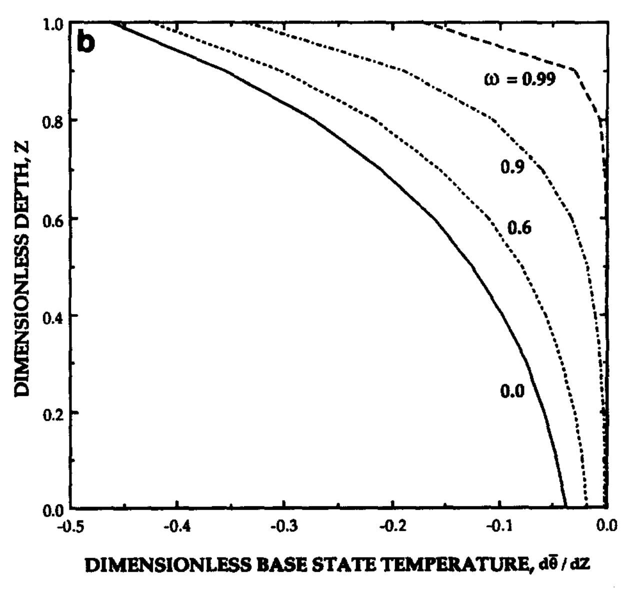

Figure 3b Scattering effect on the base-state temperature distribution for Bi = 1, Pl = 1, $\eta = 1$, $\tau = 1$, $\varepsilon_1 = 1$, and $\tau_{t2} = 1$

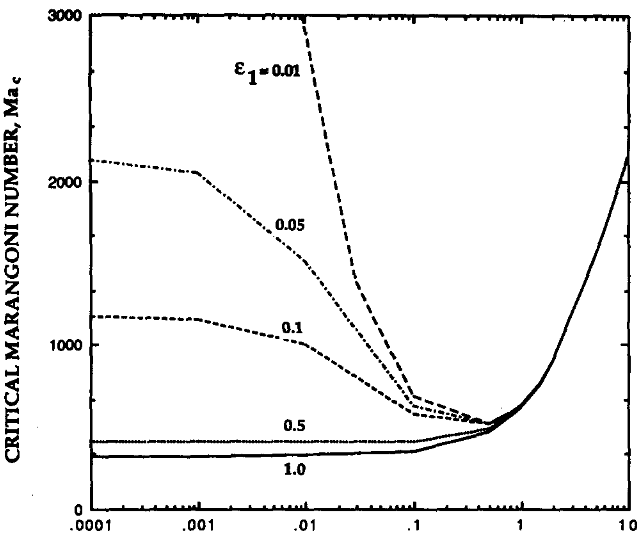

Figure 4 Effect of the lower-boundary surface properties on the critical Marangoni number for $\omega = 0$, Bi = 1, Pl = 1, $\eta = 1$, Ra = 0, and $\tau_{t2} = 1$

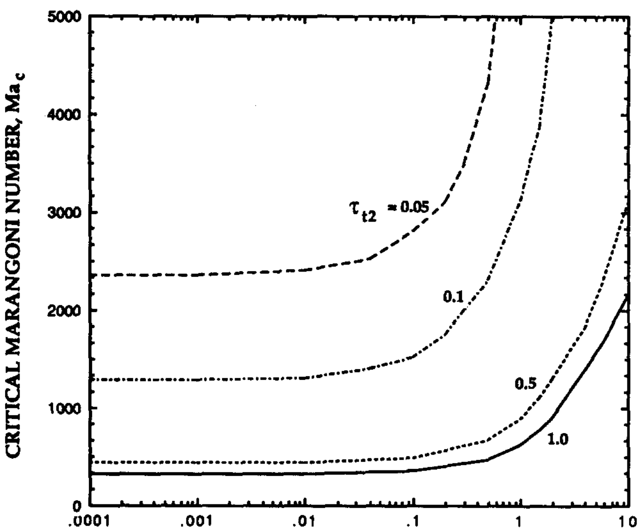

Figure 5 Effect of the upper-boundary surface properties on the critical Marangoni number for $\omega = 0$, Bi = 1, Pl = 1, $\eta = 1$, Ra = 0, and $\varepsilon_1 = 1$

(τ_{t2}) for the upper boundary. Boundaries of unity transmissivity yield the lowest critical values of the Marangoni number. In this case all the incoming radiation energy on the top surface is transmitted into the medium and none is reflected. This provides the maximum rate of heat generation within the system, thus inducing a higher degree of instability. As the transmissivity decreases, less energy is allowed to enter the medium, which in turn lessens the rate of heat generation within the medium and therefore stabilizing the system.

The effect of the gravitational force is shown in Figure 6. The results are presented to determine the effect of the Rayleigh number Ra on the critical Marangoni number. The Rayleigh number is a measure of the buoyancy force and the viscous force, and the Marangoni number represents the ratio of the surface-tension gradient to viscous force. The ratio of these two dimensionless numbers provides an estimate of the relative magnitude of the surface tension and the buoyancy forces. The surface tension force becomes dominant compared to the buoyancy force in a microgravity environment. The relationship between the Rayleigh number and the Marangoni number is shown in Figure 6, where it is seen that the Marangoni number increases with decreasing Rayleigh number. Evidently, the buoyancy force and the viscous force play a very significant role in space-based convective instability experiments. As the gravitational force increases, the system becomes less stable.

The effect of radiation scattering on the critical Marangoni number is examined and the results are plotted in Figure 7. For $\tau < 0.01$, the effect of scattering is minimal for which the fluid layer simulates a nonparticipating system. As the medium optical thickness increases further, the scattering effect becomes

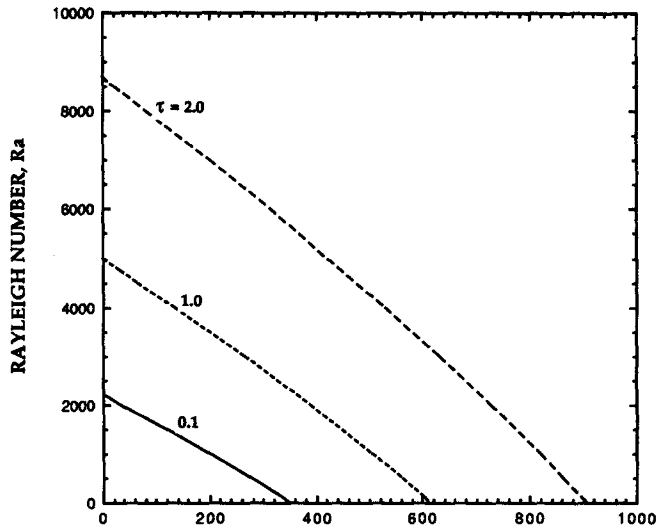

Figure 6 Effect of the gravitational force on the critical Marangoni number for Bi = 1, $\eta = 1$, Pl = 1, $\omega = 0$, $\varepsilon_1 = 1$, and $\tau_{t2} = 1$

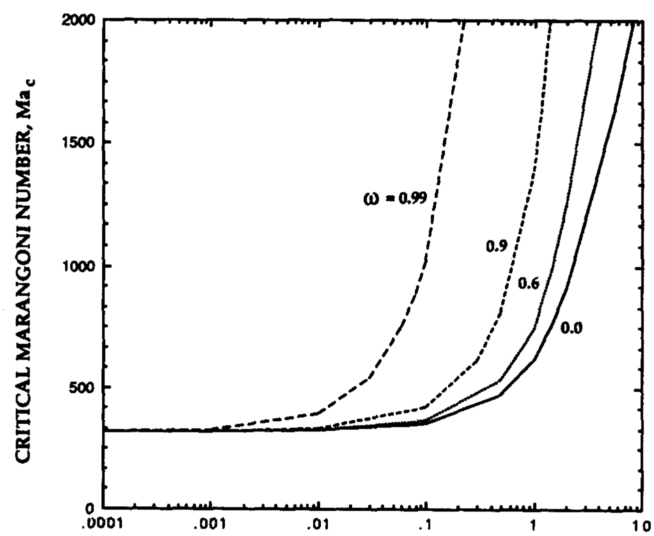

Figure 7 The effect of the scattering albedo on the critical Marangoni number for Bi = 1, Pl = 1, $\eta = 1$, Ra = 0, $\varepsilon_1 = 1$, and $\tau_{t2} = 1$

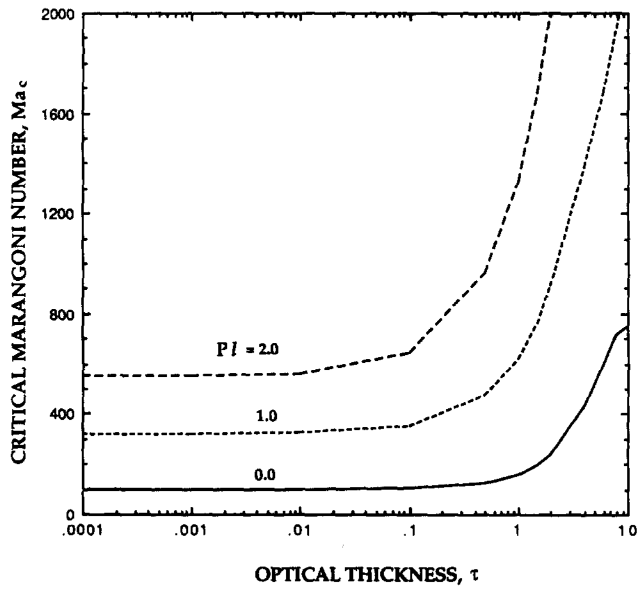

Figure 8 Effect of the Planck number on the critical Marangoni number for $Bi = 1$, $\eta = 1$, $\omega = 0$, $Ra = 0$, $\varepsilon_l = 1$, and $\tau_{t2} = 1$

a dominant factor. For a combination of large ω and τ, due to a decrease in absorption and less energy penetration, a stabilizing temperature distribution occurs throughout the layer and the system becomes more stable. Thus an increase in ω increases the stability of the system for large τ.

Finally, the effect of thermal radiation (in terms of the Planck number, Pl) is demonstrated in Figure 8. The Planck number is defined as the ratio of radiation to conduction. From a close examination of Equation 22, the slope of the dimensionless base-state temperature distribution, $d\theta/dZ$, can be lessened as thermal radiation becomes dominant within the fluid layer for $Pl > 1$; as a result, it delays the onset of instability. Therefore, the critical Marangoni number increases with increasing Planck number, which indicates that thermal radiation possesses a stabilizing effect on convection.

Concluding remarks

The conditions leading to the onset of thermocapillary convective instability in a horizontal radiation participating medium subjected to an external thermal radiative heat flux at the upper surface have been determined by using a numerical optimization technique known as the sequential gradient-restoration algorithm. The dependence of the stability characteristics on thermal radiation, the optical thickness, the surface optical properties, the gravitational forces, and the scattering albedo are investigated. In this study, the upper free surface is assumed flat as a first approximation. Although this model is an idealized one, it serves to identify the importance of thermocapillarity in the form of finite-wave and stationary mode. However, from a practical point of view, as the free-surface deformation is allowed and included, the oscillatory mode convection may play an important role in the present system. Further study is necessary to clarify their roles in thermal convective instability.

References

Arpaci, V. S. and Gozum, D. 1973. Thermal stability of radiating fluids: The Benard problem. *Phys. Fluids*, **16**, 581–588

Bayazitoglu, Y. and Lam, T. T. 1987. Marangoni convection in radiating fluids. *ASME J. Heat Transfer*, **109**, 717–722

Castillo, J. L. and Velarde, M. G. 1982. Buoyancy-thermocapillary instability: The role of interfacial deformation in one- and two-component fluid layers heated from below and above. *J. Fluid Mech.*, **125**, 463–474

Chandrasekhar, S. 1961. *Hydrodynamic and Hydromagnetic Stability*. Clarendon Press, London

Christophorides, C. and Davis, S. H. 1970. Thermal instability with radiative transfer. *Phys. Fluids*, **13**, 222–226

Davis, S. H. 1969. On the principle of exchange of stabilities. *Proc. R. Soc.*, **A310**, 341–358

Gonzalez, S. and Miele, A. 1978. Sequential gradient-restoration algorithm for optimal control problems with general boundary conditions. *J. Optimiz. Theory Appl.*, **26**, 395–425

Goody, R. M. 1956. The influence of radiative transfer on cellular convection. *J. Fluid Mech.*, **2**, 424–435

Khosla, R. K. and Murgai, M. P. 1963. A study of the combined effects of thermal radiative transfer and rotation on the gravitational stability of a hot fluid. *J. Fluid Mech.*, **16**, 97–107

Lam, T. T. and Bayazitoglu, Y. 1986a. Solution to the Orr–Sommerfeld equation for liquid film flowing down an inclined plane: An optimal approach. *Int. J. Numer. Meth. Fluids*, **6**, 883–894

Lam, T. T. and Bayazitoglu, Y. 1986b. Application of the sequential gradient-restoration algorithm to thermal convective instability problems. *J. Optimiz. Theory Appl.*, **49**, 47–63

Lam, T. T. and Bayazitoglu, Y. 1987. Effects of internal heat generation and variable viscosity on Marangoni convection. *Numer. Heat Transfer*, **11**, 165–182

Lam, T. T. and Bayazitoglu, Y. 1988. Marangoni instability with non-uniform volumetric energy source due to incident radiation. *Acta Astronautica*, **17**, 31–38

Murgai, M. P. and Khosla, P. K. 1962. A study of the combined effects of thermal radiative transfer and a magnetic field on the gravitational convection of an ionized fluid. *J. Fluid Mech.*, **14**, 433–451

Spiegel, E. A. 1960. The convective instability of a radiating fluid layer. *Astrophys. J.*, **132**, 716–728

Takashima, M. 1970. Nature of the neutral state in convective instability induced by surface-tension and buoyancy. *J. Phys. Soc. Jpn.*, **28**, 810

Takashima, M. 1981. Surface tension driven instability in a horizontal liquid layer with a deformable free surface. I. Stationary convection. *J. Phys. Soc. Jpn.*, **50**, 2745–2750

Vidal, A. and Acrivos, A. 1966. Nature of the neutral state in surface-tension driven convection. *Phys. Fluids*, **9**, 615–616

Yang, W. M. 1990. Thermal instability of a fluid layer induced by radiation. *Numer. Heat Transfer*, **17**, 365–376